Kinematic Equations

- Descriptions of Motion (words $\rightarrow$ sentences)
- In more than one dimension now

$\vec{r} = (x, y)$, $\vec{v} = (v_x, v_y)$

$\vec{a} = (a_x, a_y)$

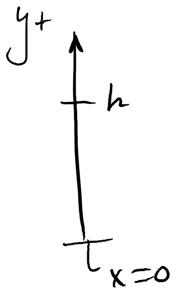
$$\vec{r} = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$x = x_i + v_{ix} t + \frac{1}{2}a_x t^2, \quad y = y_i + v_{iy} t + \frac{1}{2}a_y t^2$$

$$\vec{v} = \vec{v}_i + \vec{a}t$$

$$v_x = v_{ix} + a_x t, \quad v_y = v_{iy} + a_y t$$

Falling marbles demonstration



$a_y = -g$, $a_x = 0$

① dropped

$v_{ix} = v_{iy} = 0$

$x = 0$, $y = h$

$y = y_i + v_{iy} t - \frac{1}{2} g t^2$

$y = h - \frac{1}{2} g t^2$ → same

② shot

$v_{ix} = v_x$, $v_{iy} = 0$

$x = 0$, $y = h$

$y = y_i + v_{iy} t - \frac{1}{2} g t^2$

$y = h - \frac{1}{2} g t^2$ → same

same → t_{fall} → same → hit table @ same time

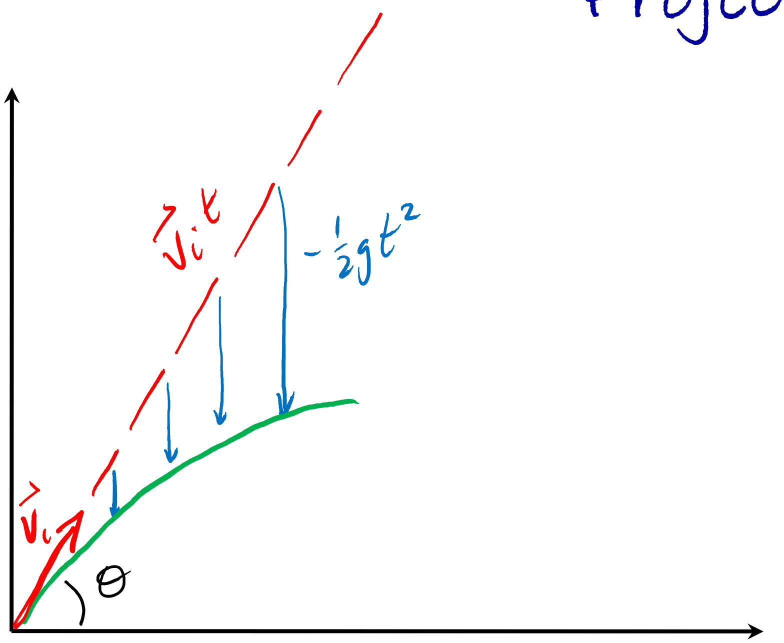
$x = \cancel{x_0}^{\,0} + \cancel{v_{ix}}^{\,0} t$

$\underline{x = 0}$

$x = \cancel{x_0}^{\,0} + v_{ix} t$

$x = v_x t_{fall}$

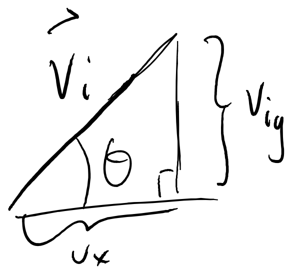
Projectile Motion



$$\vec{r} = \vec{v}_i t - \frac{1}{2}\vec{g} t^2$$

$$x = v_{ix} t \qquad a_x = 0$$

$$y = v_{iy} t - \frac{1}{2} g t^2$$



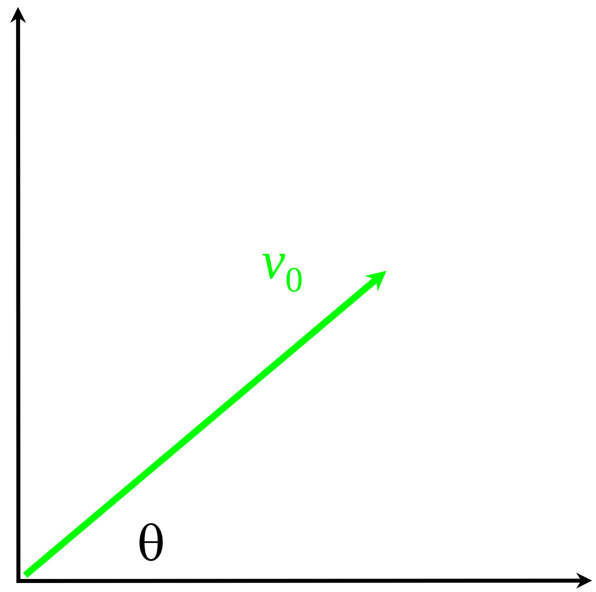
$$\sin\theta = \frac{V_{iy}}{V_i}$$

$$\cos\theta = \frac{V_{ix}}{V_i}$$

$$V_{ix} = V_i \cos\theta$$

$$V_{iy} = V_i \sin\theta$$

A proud, rejuvenated, properly-inflated football is launched with a velocity v_0 at a direction θ above the horizontal. What is its maximum height?



$$v_y = v_{iy} - gt$$

$v_y = 0 \rightarrow$ max height $\quad 0 = v_{iy} - g t_{1/2}$

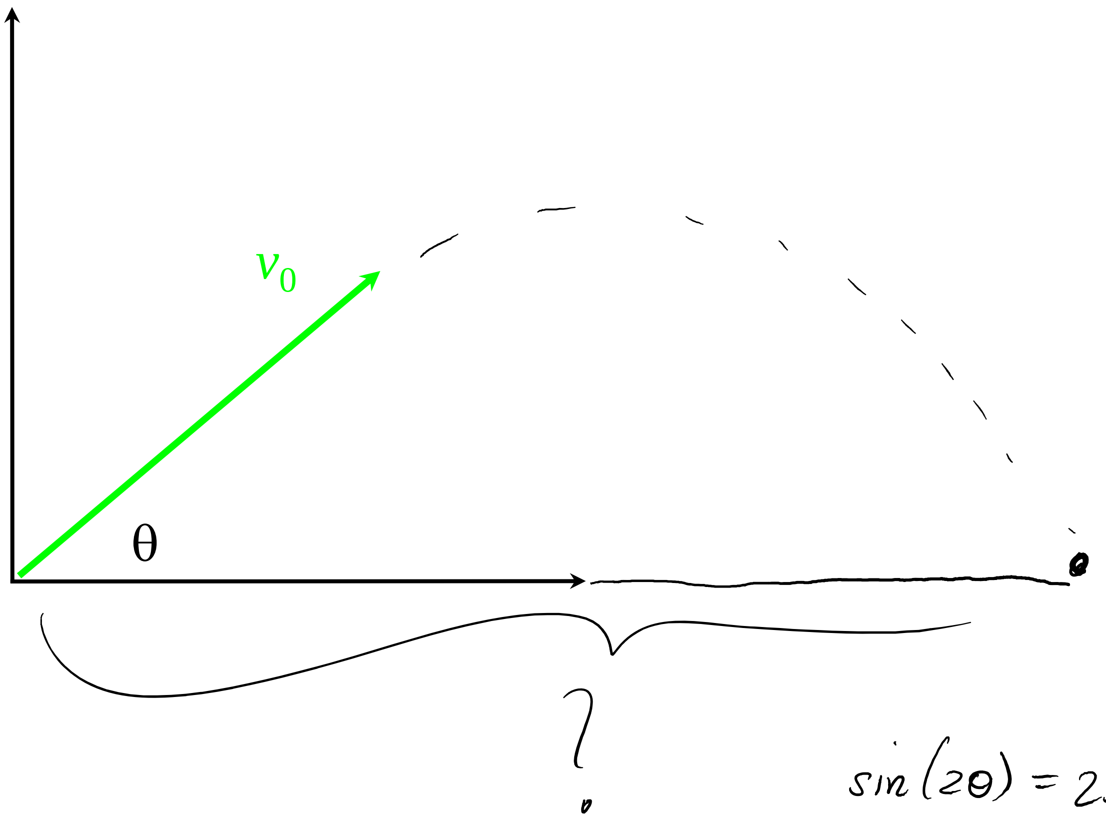
$$t_{1/2} = \frac{v_{iy}}{g} = \frac{v_i \sin\theta}{g}$$

$$y_{max} = v_{iy} t_{1/2} - \frac{1}{2} g t_{1/2}^2$$

$$= v_i \sin\theta \left(\frac{v_i \sin\theta}{g}\right) - \frac{1}{2} \cancel{g} \left(\frac{v_i^2 \sin^2\theta}{g^{\cancel{2}}}\right)$$

$$y_{max} = \frac{1}{2} \frac{v_i^2 \sin^2\theta}{g}$$

A proud, rejuvenated, properly-inflated football is launched with a velocity v_0 at a direction θ above the horizontal. What is its maximum range?



$$x = \cancel{x_i}^{\,0} + v_{ix}t + \tfrac{1}{2}\cancel{a_x}^{\,0}t^2$$

$$t_{tot} = 2 \times t_{1/2}$$

$$V_{ix} = V_0 \cos\theta$$

$$R = V_{ix}\, t_{tot} = V_0 \cos\theta \frac{2V_0 \sin\theta}{g}$$

$$= \frac{2V_0^2 \sin\theta \cos\theta}{g}$$

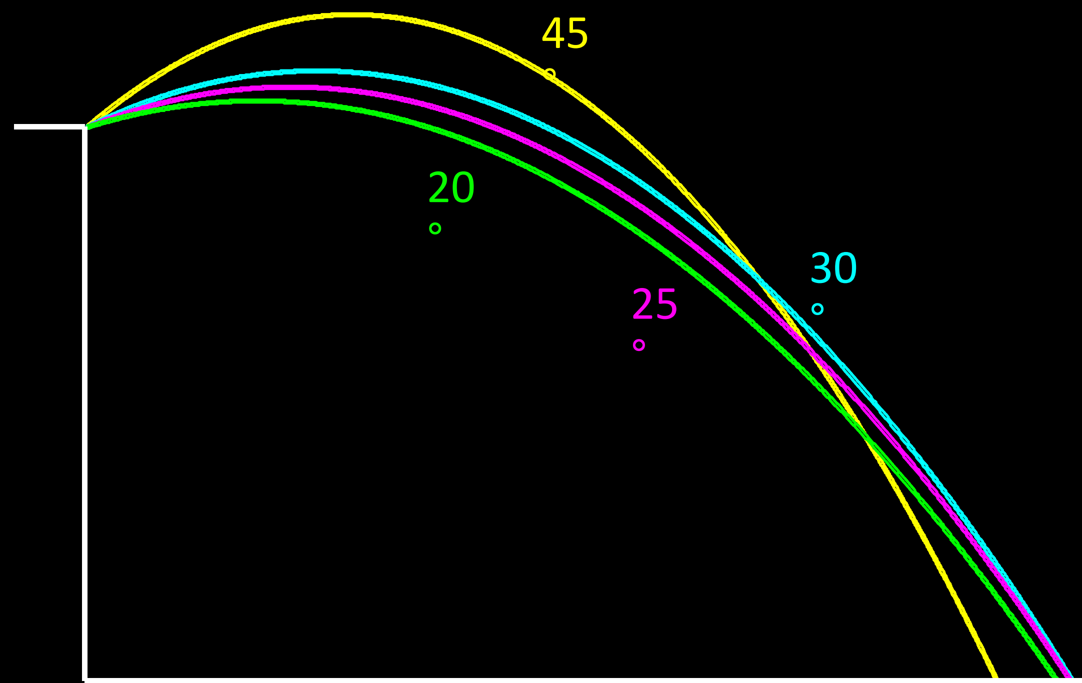
$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$R = \frac{V_0^2 \sin(2\theta)}{g}$$

Discussion of Range

$45^\circ \rightarrow$ max range

Range from a cliff:



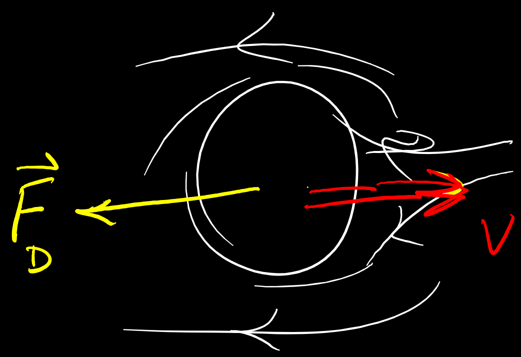
Forces

F → one object presses on another

$\vec{F} = m\vec{a}$, $\vec{a} = \dfrac{\vec{F}}{m}$ } "why" of motion

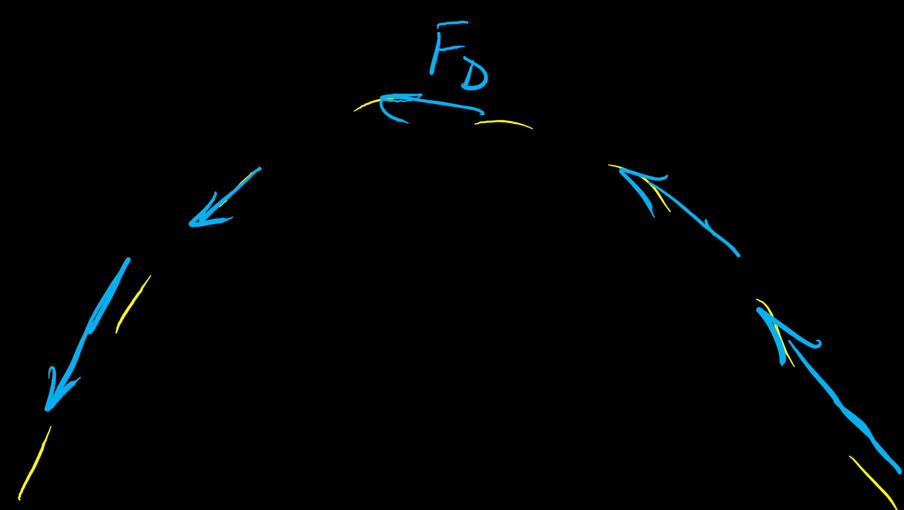
Newton's 3rd Law → $\vec{F}_A = -\vec{F}_B$

Drag Forces



$$\vec{F}_D = -b\vec{v}$$

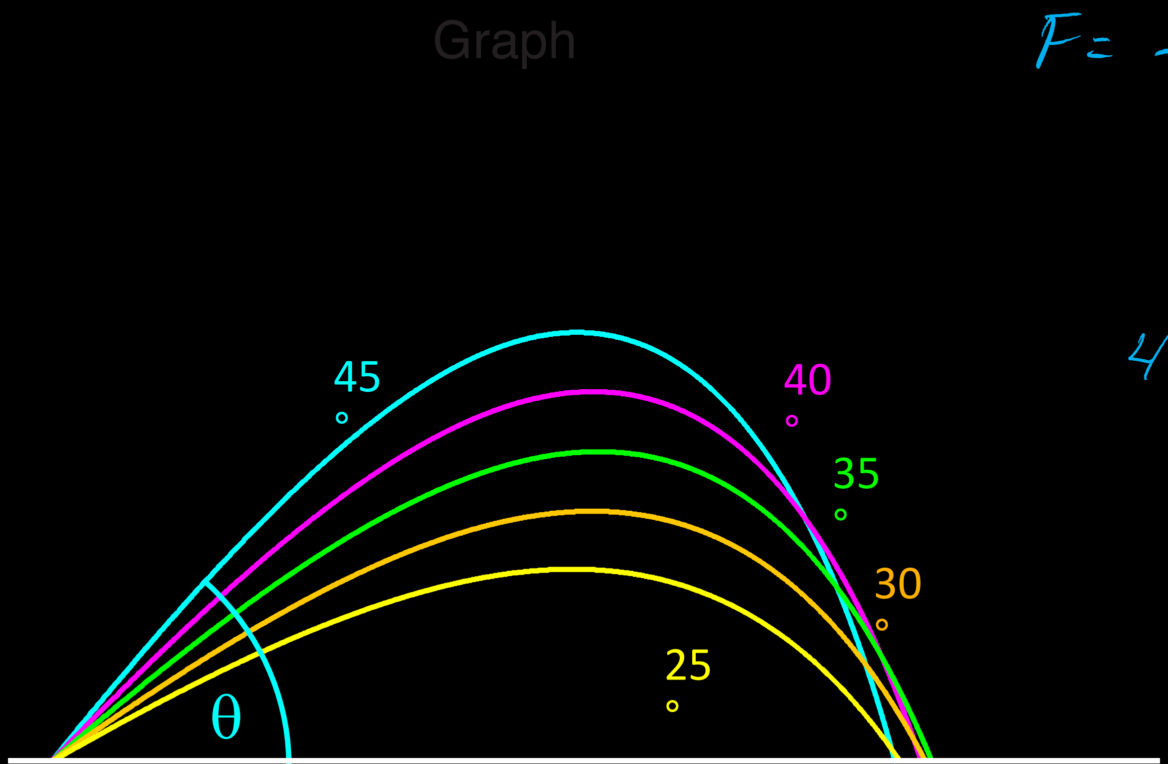
$$= -b|\vec{v}|^2$$



equations are messy

→ numerical solutions

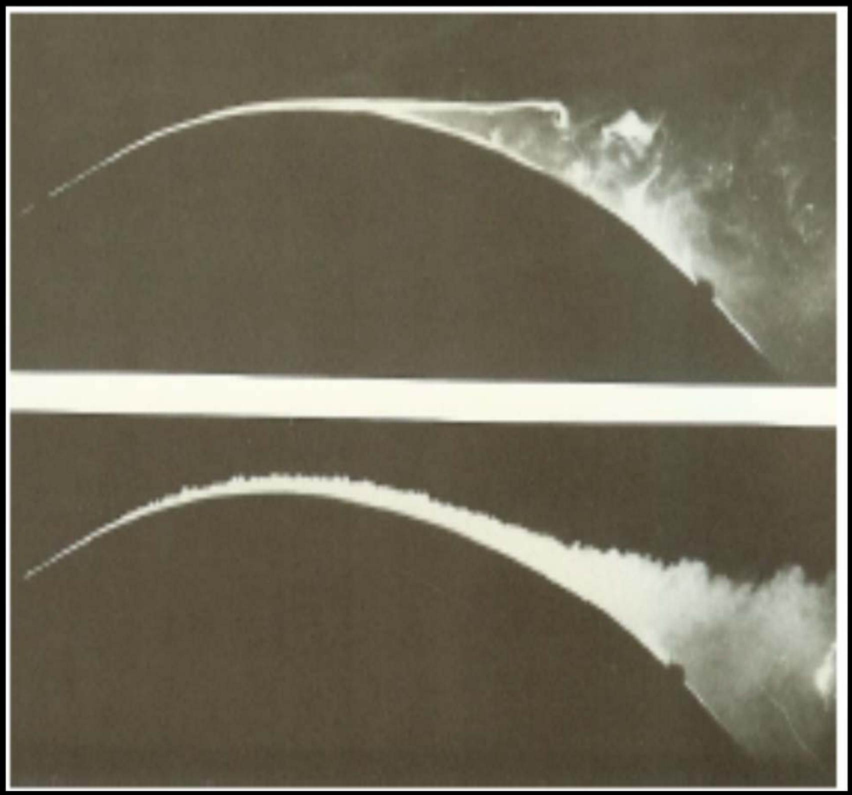
Trajectories under the influence of a drag force



$F = -(0.2)v^2$

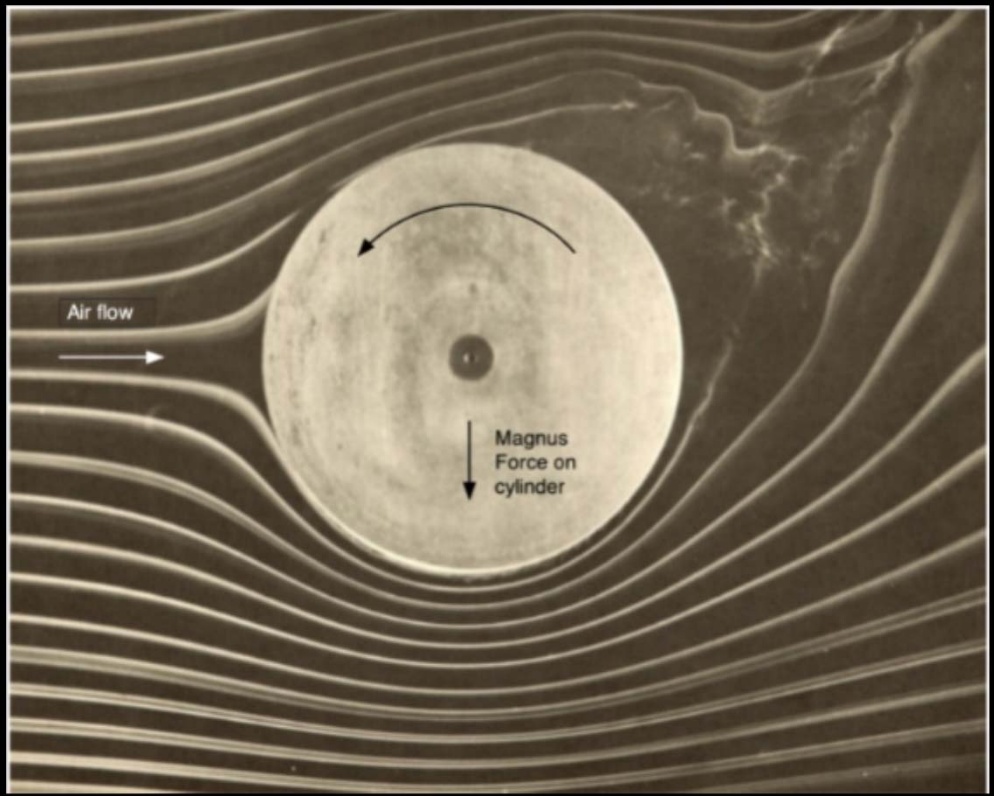
45^0 is **not** max Range

Coandă Effect



Levitating Ping-pong ball

Magnus Effect: Drag & Spin



Magnus Effect: Drag & Spin